9.3 The Converse of the Pythagorean Theorem

What you should learn

GOAL 1 Use the Converse of the Pythagorean Theorem to solve problems.

GOAL 2 Use side lengths to classify triangles by their angle measures.

Why you should learn it

▼ To determine whether **real-life** angles are right angles, such as the four angles formed by the foundation of a building in **Example 3**.

REAL LIFE

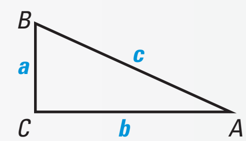
GOAL 1 USING THE CONVERSE

In Lesson 9.2, you learned that if a triangle is a right triangle, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. The Converse of the Pythagorean Theorem is also true, as stated below. Exercise 43 asks you to prove the Converse of the Pythagorean Theorem.

THEOREM

THEOREM 9.5 ***Converse of the Pythagorean Theorem***

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.



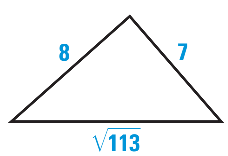
If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.

You can use the Converse of the Pythagorean Theorem to verify that a given triangle is a right triangle, as shown in Example 1.

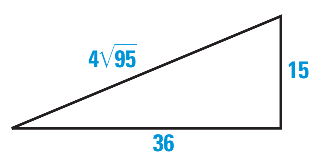
EXAMPLE 1 *Verifying Right Triangles*

The triangles below appear to be right triangles. Tell whether they are right triangles.

a.



b.



SOLUTION

Let c represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

a. $(\sqrt{113})^2 \stackrel{?}{=} 7^2 + 8^2$

$113 \stackrel{?}{=} 49 + 64$

$113 = 113$ ✓

The triangle is a right triangle.

b. $(4\sqrt{95})^2 \stackrel{?}{=} 15^2 + 36^2$

$4^2 \cdot (\sqrt{95})^2 \stackrel{?}{=} 15^2 + 36^2$

$16 \cdot 95 \stackrel{?}{=} 225 + 1296$

$1520 \neq 1521$

The triangle is not a right triangle.

GOAL 2 CLASSIFYING TRIANGLES

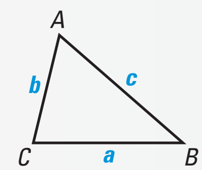
Sometimes it is hard to tell from looking whether a triangle is obtuse or acute. The theorems below can help you tell.

THEOREMS

THEOREM 9.6

If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is acute.

If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.

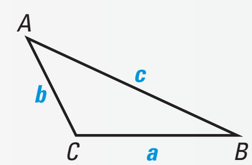


$c^2 < a^2 + b^2$

THEOREM 9.7

If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle is obtuse.

If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.



$c^2 > a^2 + b^2$

EXAMPLE 2 *Classifying Triangles*

Decide whether the set of numbers can represent the side lengths of a triangle. If they can, classify the triangle as *right*, *acute*, or *obtuse*.

a. 38, 77, 86

b. 10.5, 36.5, 37.5

SOLUTION

You can use the Triangle Inequality to confirm that each set of numbers can represent the side lengths of a triangle.

Compare the square of the length of the longest side with the sum of the squares of the lengths of the two shorter sides.

a.

$c^2 \underline{?} a^2 + b^2$	**Compare c^2 with $a^2 + b^2$.**
$86^2 \underline{?} 38^2 + 77^2$	**Substitute.**
$7396 \underline{?} 1444 + 5929$	**Multiply.**
$7396 > 7373$	**c^2 is greater than $a^2 + b^2$.**

▶ Because $c^2 > a^2 + b^2$, the triangle is obtuse.

b.

$c^2 \underline{?} a^2 + b^2$	**Compare c^2 with $a^2 + b^2$.**
$37.5^2 \underline{?} 10.5^2 + 36.5^2$	**Substitute.**
$1406.25 \underline{?} 110.25 + 1332.25$	**Multiply.**
$1406.25 < 1442.5$	**c^2 is less than $a^2 + b^2$.**

▶ Because $c^2 < a^2 + b^2$, the triangle is acute.

STUDENT HELP

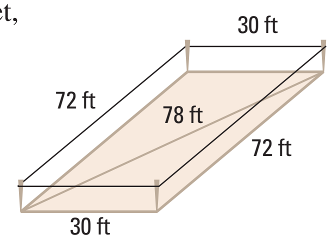
Look Back
For help with the Triangle Inequality, see p. 297.

EXAMPLE 3 *Building a Foundation*

CONSTRUCTION You use four stakes and string to mark the foundation of a house. You want to make sure the foundation is rectangular.

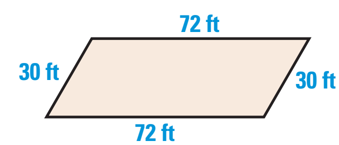
a. A friend measures the four sides to be 30 feet, 30 feet, 72 feet, and 72 feet. He says these measurements prove the foundation is rectangular. Is he correct?

b. You measure one of the diagonals to be 78 feet. Explain how you can use this measurement to tell whether the foundation will be rectangular.



SOLUTION

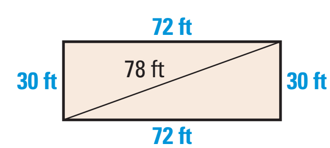
a. Your friend is not correct. The foundation could be a nonrectangular parallelogram, as shown at the right.



STUDENT HELP

Look Back
For help with classifying quadrilaterals, see Chapter 6.

b. The diagonal divides the foundation into two triangles. Compare the square of the length of the longest side with the sum of the squares of the shorter sides of one of these triangles. Because $30^2 + 72^2 = 78^2$, you can conclude that both the triangles are right triangles.



▶ The foundation is a parallelogram with two right angles, which implies that it is rectangular.

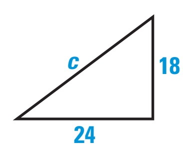
GUIDED PRACTICE

Vocabulary Check ✓

1. State the Converse of the Pythagorean Theorem in your own words.

Concept Check ✓

2. Use the triangle shown at the right. Find values for c so that the triangle is acute, right, and obtuse.



Skill Check ✓

In Exercises 3–6, match the side lengths with the appropriate description.

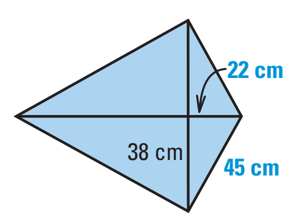
3. 2, 10, 11 **A.** right triangle

4. 13, 5, 7 **B.** acute triangle

5. 5, 11, 6 **C.** obtuse triangle

6. 6, 8, 10 **D.** not a triangle

7. **KITE DESIGN** You are making the diamond-shaped kite shown at the right. You measure the crossbars to determine whether they are perpendicular. Are they? Explain.



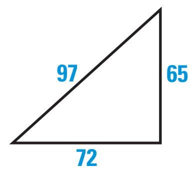
PRACTICE AND APPLICATIONS

STUDENT HELP

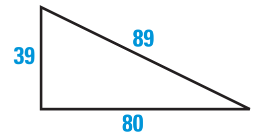
Extra Practice to help you master skills is on pp. 819 and 820.

VERIFYING RIGHT TRIANGLES **Tell whether the triangle is a right triangle.**

8.



9.



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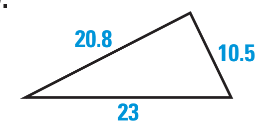

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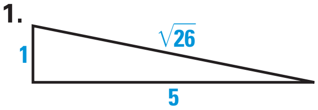

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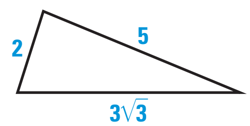
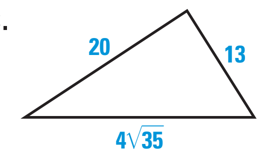


13.



CLASSIFYING TRIANGLES **Decide whether the numbers can represent the side lengths of a triangle. If they can, classify the triangle as *right, acute,* or *obtuse.***

14. 20, 99, 101 **15.** 21, 28, 35 **16.** 26, 10, 17

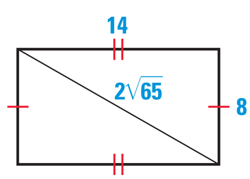
17. 2, 10, 12 **18.** 4, $\sqrt{67}$, 9 **19.** $\sqrt{13}$, 6, 7

20. 16, 30, 34 **21.** 10, 11, 14 **22.** 4, 5, 5

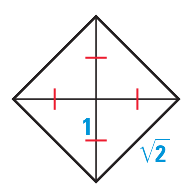
23. 17, 144, 145 **24.** 10, 49, 50 **25.** $\sqrt{5}$, 5, 5.5

CLASSIFYING QUADRILATERALS **Classify the quadrilateral. Explain how you can prove that the quadrilateral is that type.**

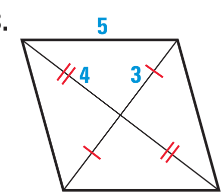
26.



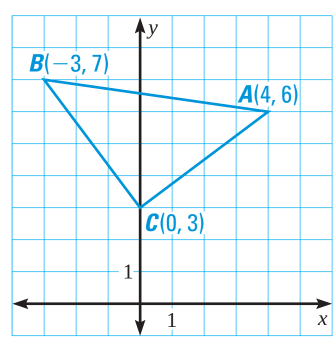
27.



28.



CHOOSING A METHOD **In Exercises 29–31, you will use two different methods for determining whether $\triangle ABC$ is a right triangle.**



29. ***Method 1*** Find the slope of $\overline{AC}$ and the slope of $\overline{BC}$. What do the slopes tell you about $\angle ACB$? Is $\triangle ABC$ a right triangle? How do you know?

30. ***Method 2*** Use the Distance Formula and the Converse of the Pythagorean Theorem to determine whether $\triangle ABC$ is a right triangle.

31. Which method would you use to determine whether a given triangle is right, acute, or obtuse? Explain.

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 8–13, 30
Example 2: Exs. 14–28, 31–35
Example 3: Exs. 39, 40

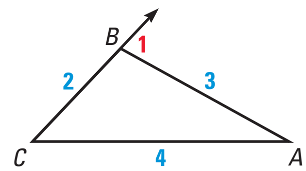
USING ALGEBRA **Graph points *P, Q,* and *R*. Connect the points to form $\triangle PQR$. Decide whether $\triangle PQR$ is *right, acute,* or *obtuse.***

32. $P(-3, 4)$, $Q(5, 0)$, $R(-6, -2)$ **33.** $P(-1, 2)$, $Q(4, 1)$, $R(0, -1)$

PROOF **Write a proof.**

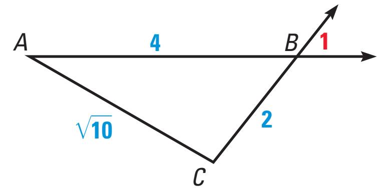
34. GIVEN ▶ $AB = 3, BC = 2,$
$AC = 4$

PROVE ▶ $\angle 1$ is acute.



35. GIVEN ▶ $AB = 4, BC = 2,$
$AC = \sqrt{10}$

PROVE ▶ $\angle 1$ is acute.



STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with Ex. 36.

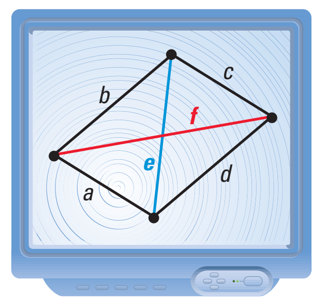
36. PROOF Prove that if a, b, and c are a Pythagorean triple, then ka, kb, and kc (where $k > 0$) represent the side lengths of a right triangle.

37. PYTHAGOREAN TRIPLES Use the results of Exercise 36 and the Pythagorean triple 5, 12, 13. Which sets of numbers can represent the side lengths of a right triangle?

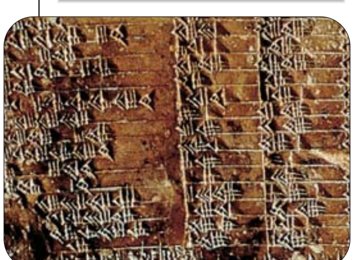
A. 50, 120, 130 **B.** 20, 48, 56 **C.** $1\frac{1}{4}, 3, 3\frac{1}{4}$ **D.** 1, 2.4, 2.6

38. TECHNOLOGY Use geometry software to construct each of the following figures: a nonspecial quadrilateral, a parallelogram, a rhombus, a square, and a rectangle. Label the sides of each figure a, b, c, and d. Measure each side. Then draw the diagonals of each figure and label them e and f. Measure each diagonal. For which figures does the following statement appear to be true?

$$a^2 + b^2 + c^2 + d^2 = e^2 + f^2$$



FOCUS ON APPLICATIONS



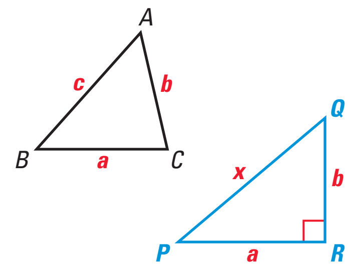
REAL LIFE BABYLONIAN TABLET
This photograph shows part of a Babylonian clay tablet made around 350 B.C. The tablet contains a table of numbers written in cuneiform characters.

39. HISTORY CONNECTION The Babylonian tablet shown at the left contains several sets of triangle side lengths, suggesting that the Babylonians may have been aware of the relationships among the side lengths of right, triangles. The side lengths in the table at the right show several sets of numbers from the tablet. Verify that each set of side lengths forms a Pythagorean triple.

a	b	c
120	119	169
4,800	4,601	6,649
13,500	12,709	18,541

40. AIR TRAVEL You take off in a jet from Cincinnati, Ohio, and fly 403 miles due east to Washington, D.C. You then fly 714 miles to Tallahassee, Florida. Finally, you fly 599 miles back to Cincinnati. Is Cincinnati directly north of Tallahassee? If not, how would you describe its location relative to Tallahassee?

41. **DEVELOPING PROOF** Complete the proof of Theorem 9.6 on page 544.



GIVEN ▶ In $\triangle ABC$, $c^2 < a^2 + b^2$.

PROVE ▶ $\triangle ABC$ is an acute triangle.

Plan for Proof Draw right $\triangle PQR$ with side lengths a, b, and x. Compare lengths c and x.

Statements	Reasons
1. $x^2 = a^2 + b^2$	**1.** ___?___
2. $c^2 < a^2 + b^2$	**2.** ___?___
3. $c^2 < x^2$	**3.** ___?___
4. $c < x$	**4.** A property of square roots
5. $m\angle C < m\angle R$	**5.** ___?___
6. $\angle C$ is an acute angle.	**6.** ___?___
7. $\triangle ABC$ is an acute triangle.	**7.** ___?___

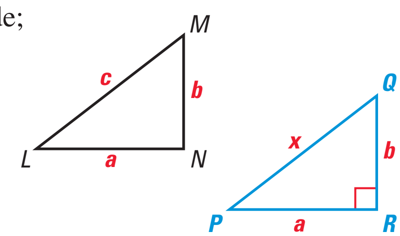
42. **PROOF** Prove Theorem 9.7 on page 544. Include a diagram and *Given* and *Prove* statements. (*Hint:* Look back at Exercise 41.)

43. **PROOF** Prove the Converse of the Pythagorean Theorem.

GIVEN ▶ In $\triangle LNM$, $\overline{LM}$ is the longest side; $c^2 = a^2 + b^2$.

PROVE ▶ $\triangle LNM$ is a right triangle.

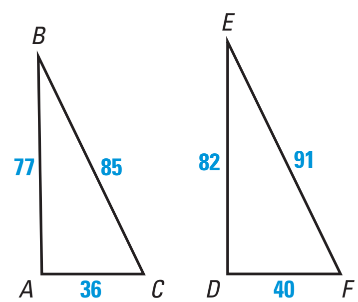
Plan for Proof Draw right $\triangle PQR$ with side lengths a, b, and x. Compare lengths c and x.



Test Preparation

QUANTITATIVE COMPARISON **Choose the statement that is true about the given quantities.**

Ⓐ The quantity in column A is greater.
Ⓑ The quantity in column B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the given information.



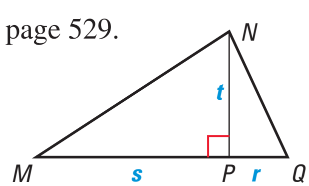
	Column A	Column B
44.	$m\angle A$	$m\angle D$
45.	$m\angle B + m\angle C$	$m\angle E + m\angle F$

★ Challenge

46. **PROOF** Prove the converse of Theorem 9.2 on page 529.

GIVEN ▶ In $\triangle MQN$, altitude $\overline{NP}$ is drawn to $\overline{MQ}$; t is the geometric mean of r and s.

PROVE ▶ $\triangle MQN$ is a right triangle.



EXTRA CHALLENGE
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MIXED REVIEW

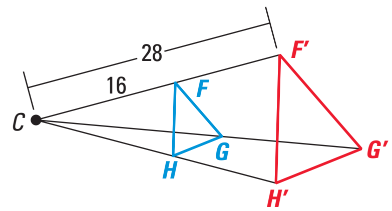
SIMPLIFYING RADICALS **Simplify the expression.** (Skills Review, p. 799, for 9.4)

47. $\sqrt{22} \cdot \sqrt{2}$ **48.** $\sqrt{6} \cdot \sqrt{8}$ **49.** $\sqrt{14} \cdot \sqrt{6}$ **50.** $\sqrt{15} \cdot \sqrt{6}$

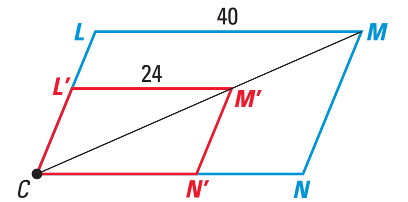
51. $\frac{3}{\sqrt{11}}$ **52.** $\frac{4}{\sqrt{5}}$ **53.** $\frac{12}{\sqrt{18}}$ **54.** $\frac{8}{\sqrt{24}}$

DILATIONS **Identify the dilation and find its scale factor.** (Review 8.7)

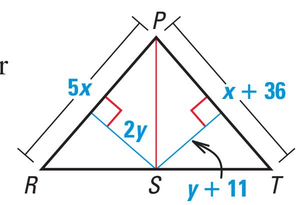
55.



56.



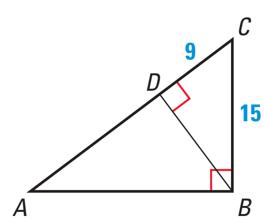
57. **USING ALGEBRA** In the diagram, $\overrightarrow{PS}$ bisects $\angle RPT$, and $\overline{PS}$ is the perpendicular bisector of $\overline{RT}$. Find the values of x and y. (Review 5.1)



QUIZ 1

Self-Test for Lessons 9.1–9.3

In Exercises 1–4, use the diagram. (Lesson 9.1)



1. Write a similarity statement about the three triangles in the diagram.

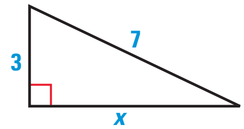
2. Which segment's length is the geometric mean of CD and AD?

3. Find AC.

4. Find BD.

Find the unknown side length. Simplify answers that are radicals. (Lesson 9.2)

5.



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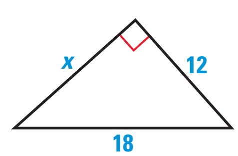

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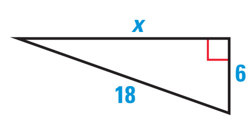

8. **CITY PARK** The diagram shown at the right shows the dimensions of a triangular city park. Does this city park have a right angle? Explain. (Lesson 9.3)